Lecture 19: Liveness analysis

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601.428/628 Compilers and Interpreters

Today

- Dataflow analysis
- Liveness analysis
- `InstructionSequence` and `ControlFlowGraph`

Dataflow Analysis

Optimization scopes

From last lecture:

Scope	Can analyze/transform	Example
Local	Single basic blocks	Local value numbering
Regional	Multiple BBs w/o control joins	Super LVN
Global	All basic blocks within a function	Liveness analysis
Interprocedural	All code in the program	Function inlining

Liveness analysis (our topic today) is a *global* analysis

It's an instance of *dataflow analysis*, an important general technique for global analysis

Dataflow Analysis

Goal of dataflow analysis: determine a *fact* at each location within a function

A dataflow fact is often a *set*

- E.g., set of registers currently containing a live value

A dataflow analysis is executed on a *control-flow graph* (graph of *basic blocks*)

The general idea is that the dataflow analysis models

- the effect of the basic block on a dataflow fact flowing through the basic block, taking into account the semantics of the instructions in the block
- the effect of control paths coming together, where it is possible for a location to be reached from multiple predecessor blocks

A dataflow analysis can be *forwards* or *backwards*

Liveness Analysis

Liveness analysis

Which storage locations (e.g., virtual registers) contain a "live" value?

"Live" means "is used on some forward path"

Very useful!

- If we see a store to a location such that the value in that location is not live, we can eliminate the store (dead store elimination)
- If we're considering a transformation that eliminates a store to a location, it's only legal if the value being stored is dead (correctness requirement for peephole rewrites)
- If any locations other than function parameters contain live values at the entry to a function, then the function could use an uninitialized value

Memory loads and stores

When a variable's storage is in memory, there could be one or more pointers pointing to it ("pointer aliasing").

That means that when a data value is stored to memory, it could potentially change the value of any memory location.

Conservative approach (always correct, but least precise):

```
... = *p;  // make no assumptions about what value is loaded

*q = ...;  // assume that any memory location might be written to
```

Because virtual registers aren't memory and don't have addresses, we don't need to worry about aliasing. Changing the value of one vreg can't change the value of any other vreg.

Liveness Analysis

Each dataflow fact is a set of locations (i.e., vregs) containing live values.

LiveOut(n) is the set of locations containing live values at the end of block n

UEVar(n) is the set of locations containing a value that is definitely used in basic block n (i.e., it appears as a source operand prior to any instruction storing to it)

VarKill(n) is the set of locations for which an instruction in block n stores a value to the location

Computation of facts

Initially: for each block n, $\text{LiveOut}(n) = \emptyset$
Iteratively:

$$\text{LiveOut}(n) = \cup_{m \in \text{succ}(n)}(\text{UEVar}(m) \cup (\text{LiveOut}(m) \cap \overline{\text{VarKill}(m)}))$$

The idea is that we start out by assuming that no vregs contain live values at the end of each block. However, for each successor m of block n

- any vreg that is definitely used in block m prior to being overwritten ($\text{UEVar}(m)$) is definitely live at the end of n
- any vreg that is live at the end of m and not in $\text{VarKill}(m)$ is also live at the end of n

Iterative computation

In general, we want to compute accurate liveness for block n's successors before we compute liveness for block n

However, the control flow graph can have cycles (e.g., if the function has a loop), so this isn't always possible

General strategy: repeatedly recompute liveness for all blocks until none of the facts change

- The LiveOut sets start out as empty; when they stop growing, we're done

Algorithm

```
-- n is number of basic blocks
for (i = 0 to n − 1)
    LiveOut(i) ← ∅
changed ← true
while changed
    changed ← false
    for (i = 0 to n − 1)
        old ← LiveOut(i)
        recompute LiveOut(i)
        if old ≠ LiveOut(i)
            changed ← true
```

Note that the same fixed-point solution is reached regardless of the order in which blocks are recomputed, but some orders are better for efficiency (more about this when we fully cover dataflow analysis.)

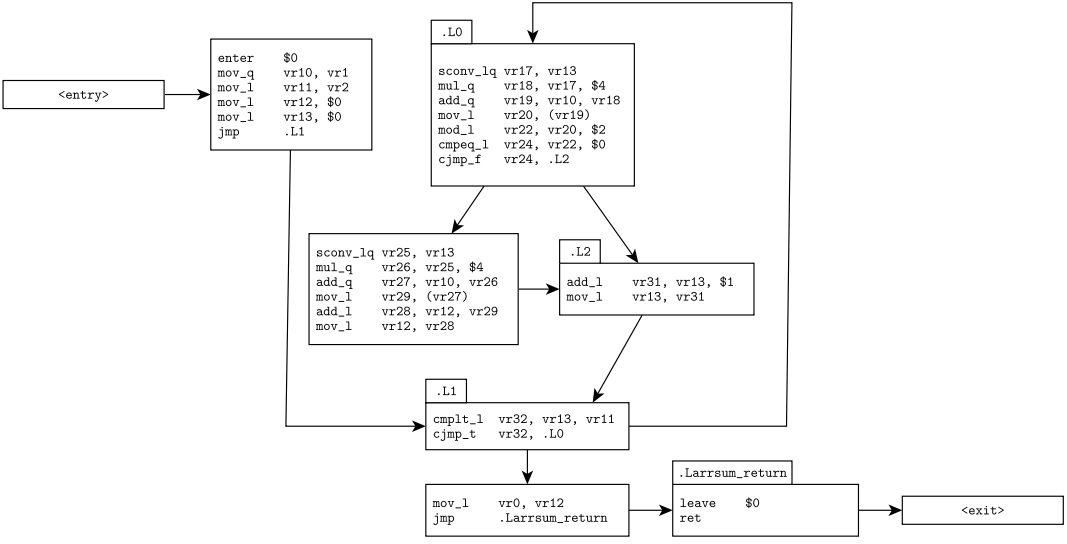
Example

```
// C code
int arrsum(int *a, int n) {
  int sum, i;
  sum = 0;
  for (i = 0; i < n; i = i + 1) {
    int elt;
    elt = a[i];
    if (elt % 2 == 0)
      sum = sum + a[i];
  }
  return sum;
}
```

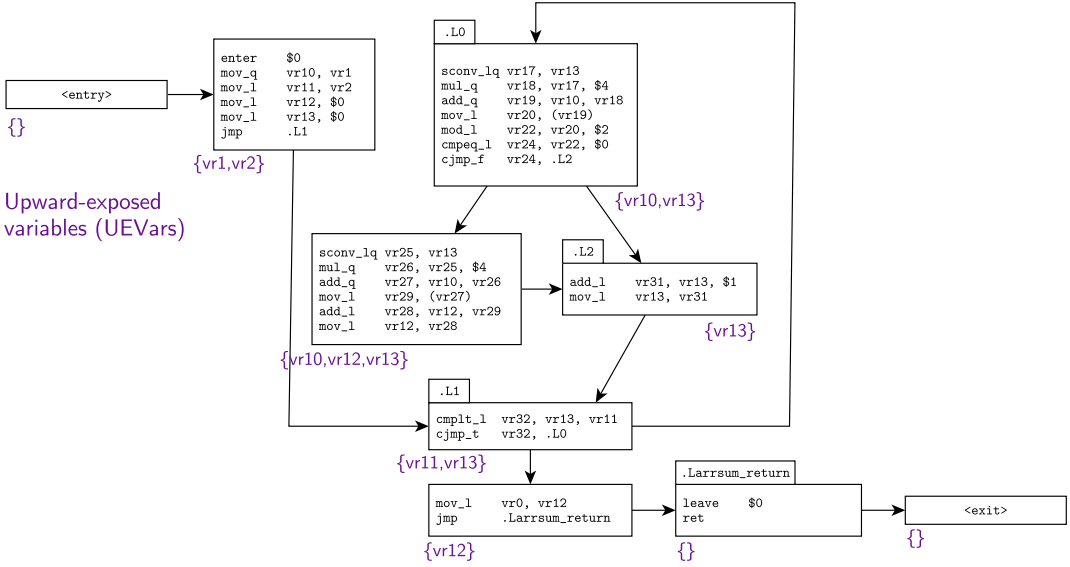
Example

```
// High-level code
// (continues on right)
        .globl arrsum
arrsum:
        enter    $0
        mov_q    vr10, vr1
        mov_l    vr11, vr2
        mov_l    vr12, $0
        mov_l    vr13, $0
        jmp      .L1
.L0:
        sconv_lq vr17, vr13
        mul_q    vr18, vr17, $4
        add_q    vr19, vr10, vr18
        mov_l    vr20, (vr19)
        mod_l    vr22, vr20, $2
        cmpeq_l  vr24, vr22, $0
        cjmp_f   vr24, .L2
        sconv_lq vr25, vr13
        mul_q    vr26, vr25, $4
        add_q    vr27, vr10, vr26
        mov_l    vr29, (vr27)
        add_l    vr28, vr12, vr29
        mov_l    vr12, vr28
.L2:
        add_l    vr31, vr13, $1
        mov_l    vr13, vr31
.L1:
        cmplt_l  vr32, vr13, vr11
        cjmp_t   vr32, .L0
        mov_l    vr0, vr12
        jmp      .Larrsum_return
.Larrsum_return:
        leave    $0
        ret
```

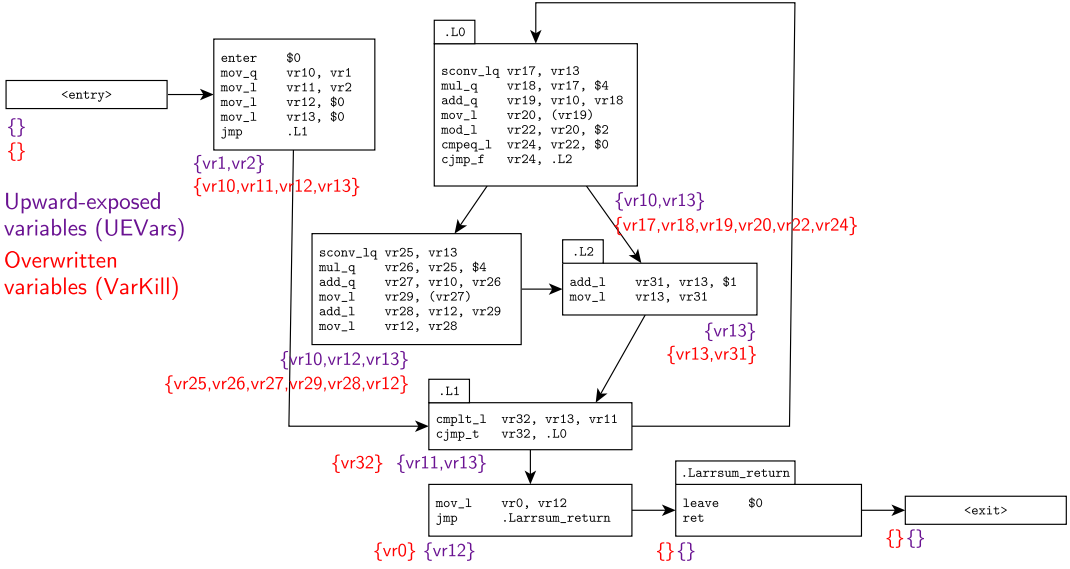
Example


<entry>
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
mov_l vr0, vr12
jmp .Larrsum_return
.Larrsum_return
leave $0
ret
<exit>

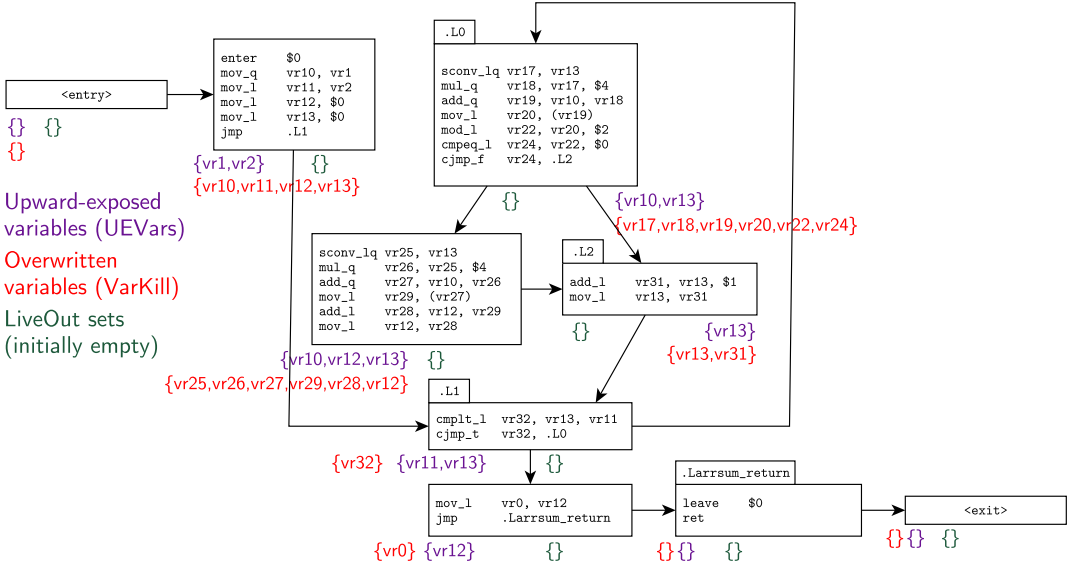
Example


<entry>
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2}
Upward-exposed variables (UEVars)
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr13}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
{vr10,vr12,vr13}
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr13}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr11,vr13}
mov_l vr0, vr12
jmp .Larrsum_return
{vr12}
.Larrsum_return
leave $0
ret
{}
<exit>
{}

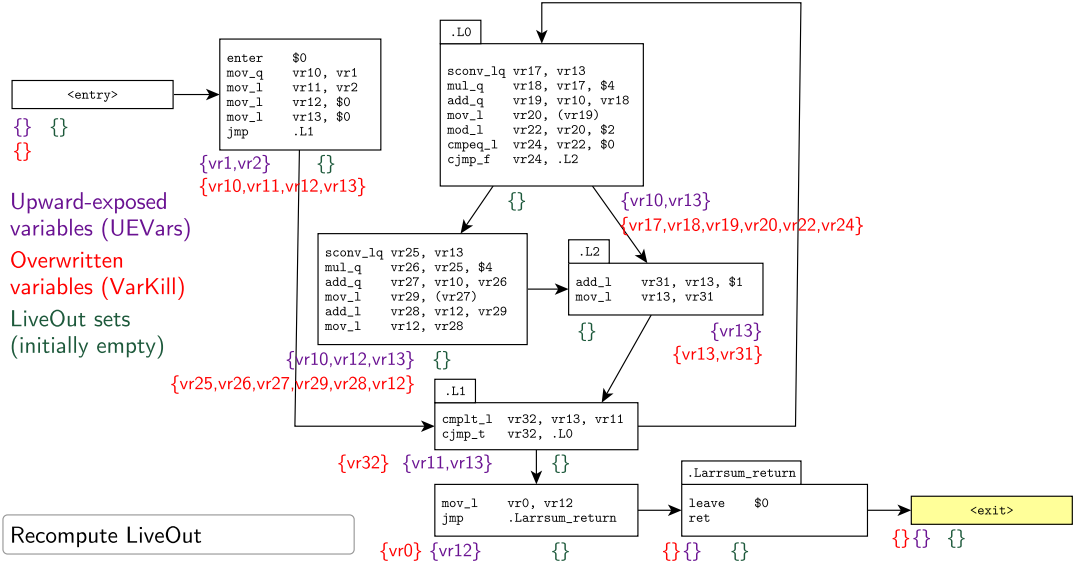
Example


<entry>
{}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2}
{vr10,vr11,vr12,vr13}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr13}
{vr13,vr31}
{vr10,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12}
{}{}
{}{}

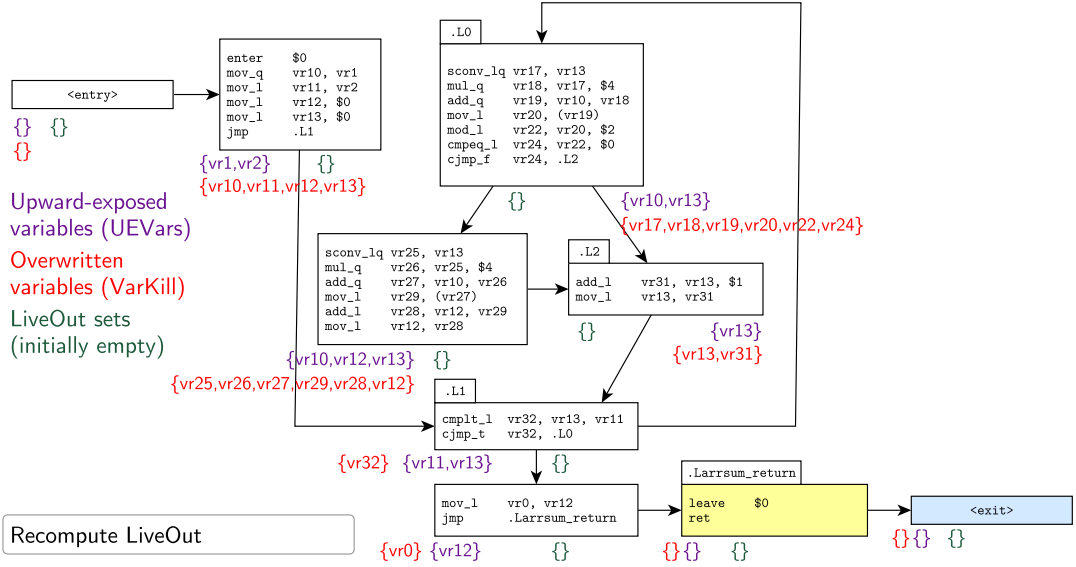
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{}
{vr13}
{vr13,vr31}
{vr10,vr12,vr13} {}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)

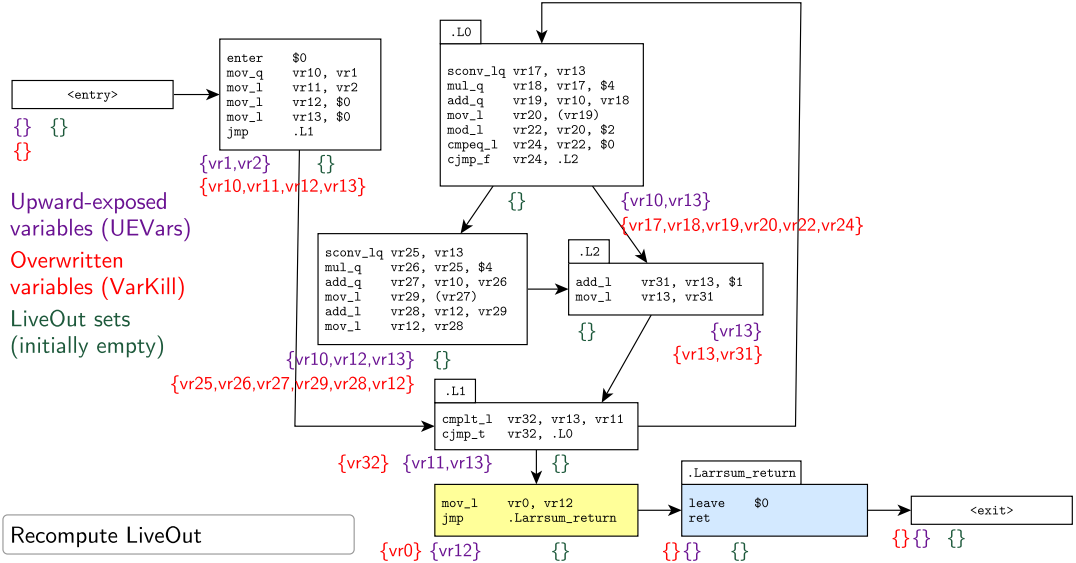
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{}
{vr13}
{vr13,vr31}
{vr10,vr12,vr13} {}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut

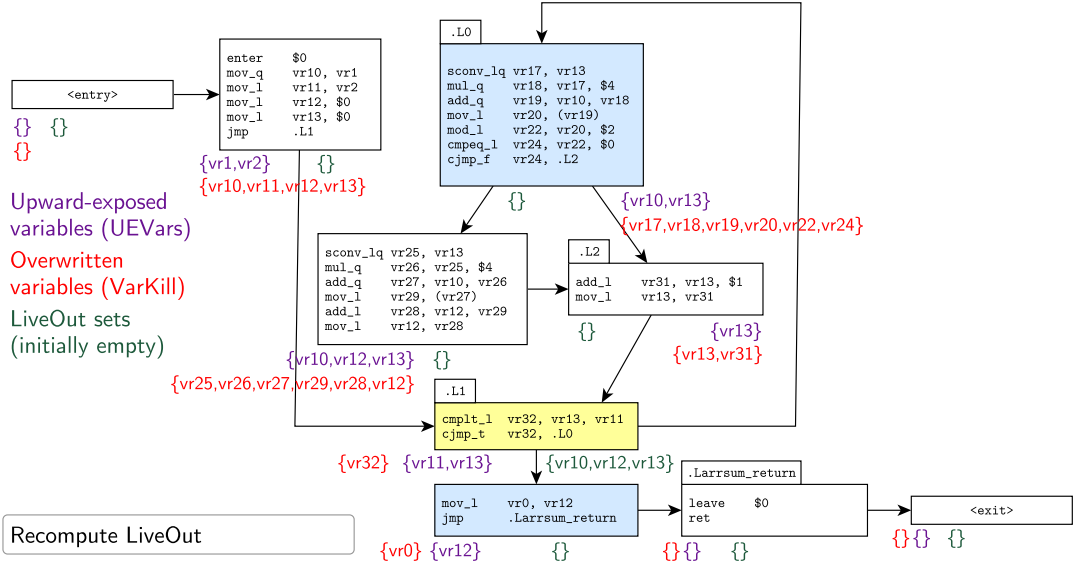
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{}
{vr13}
{vr13,vr31}
{vr10,vr12,vr13} {}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut

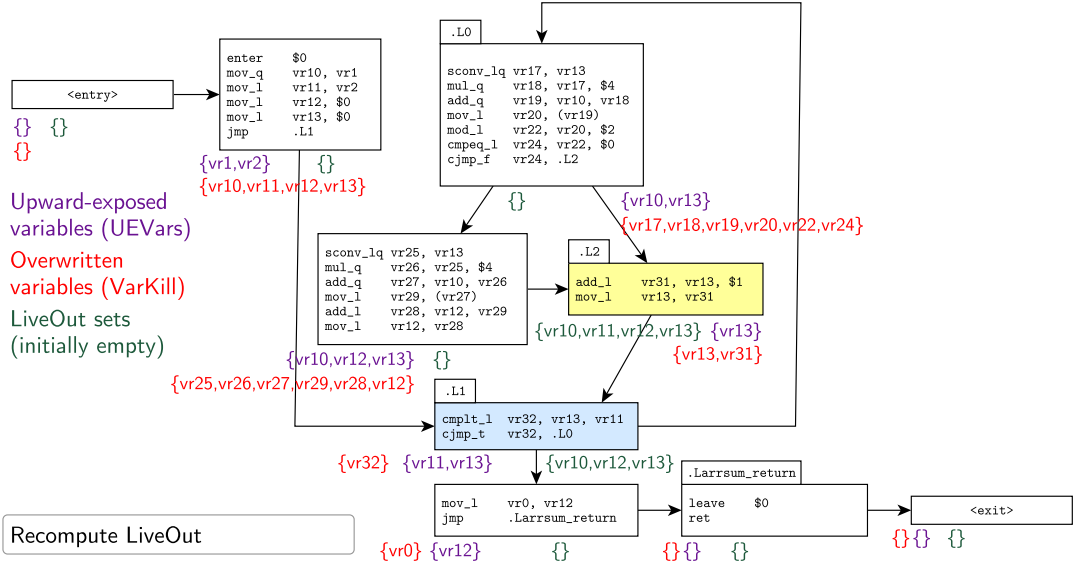
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{}
{vr13}
{vr13,vr31}
{vr10,vr12,vr13} {}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut

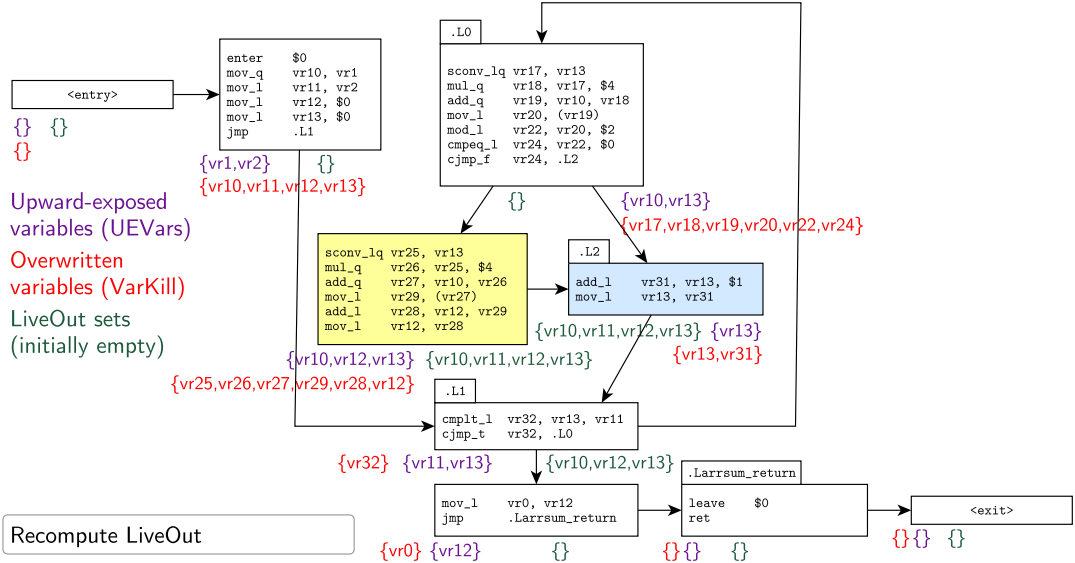
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{}
{vr13}
{vr13,vr31}
{vr10,vr12,vr13} {}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut

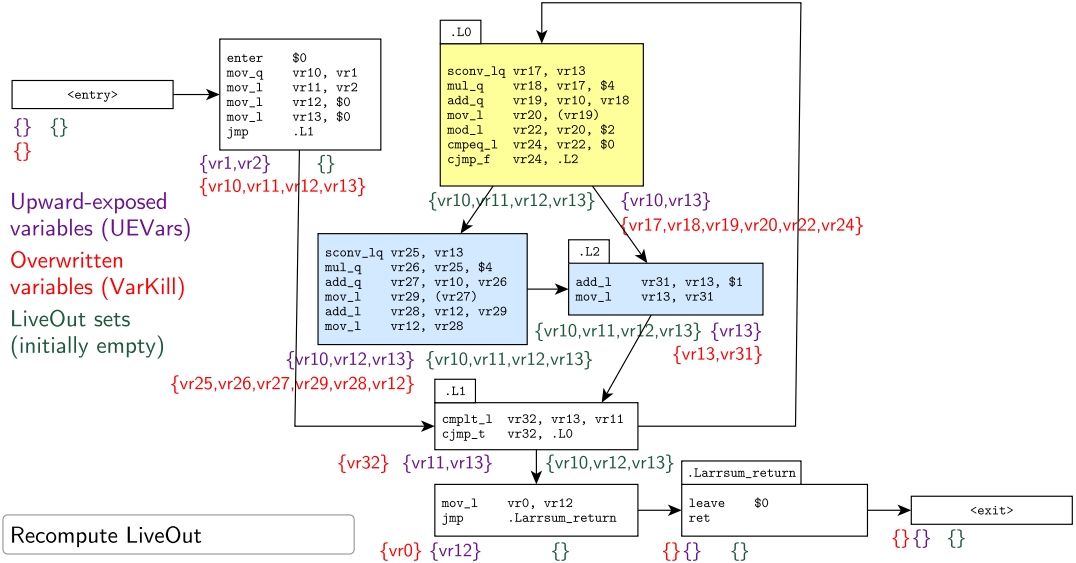
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut

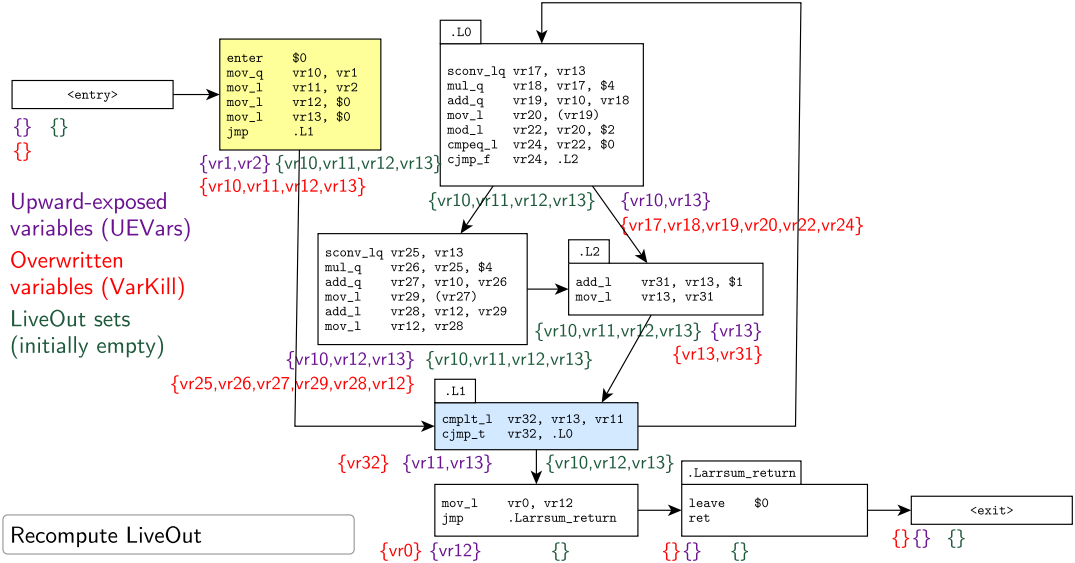
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Recompute LiveOut

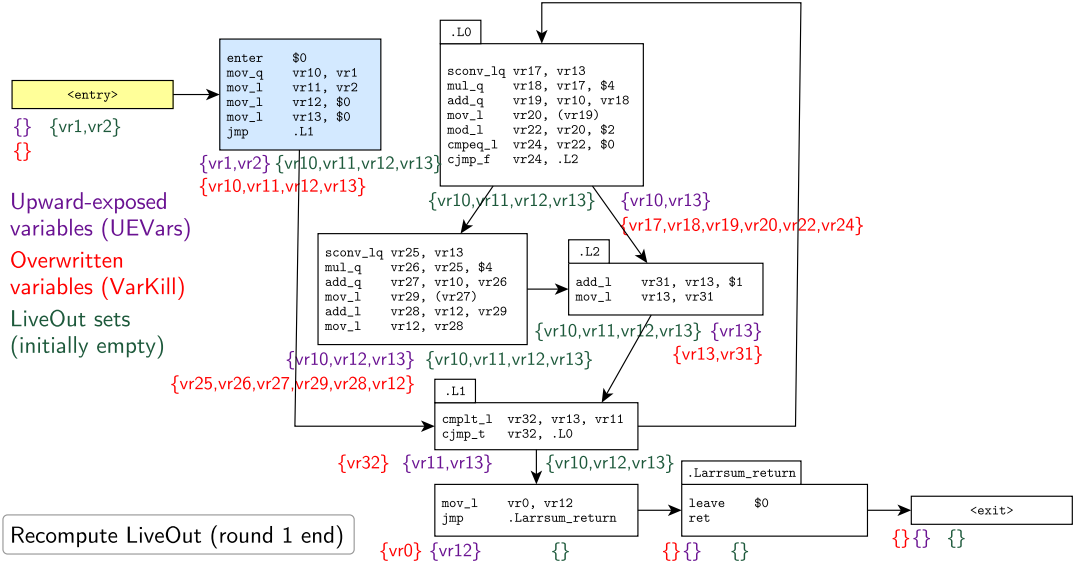
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Recompute LiveOut

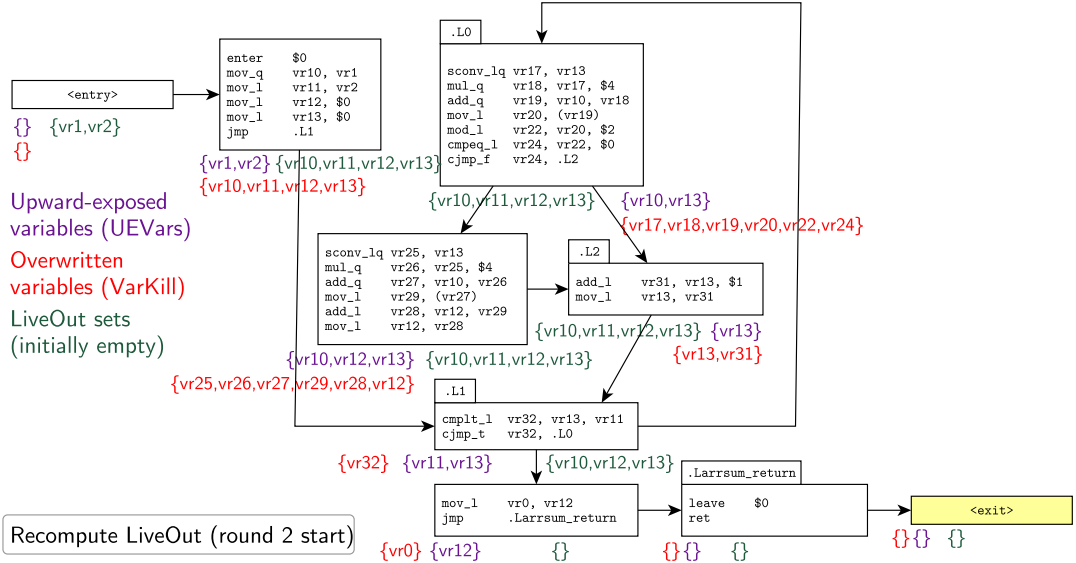
Example


<entry>
{} {}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut

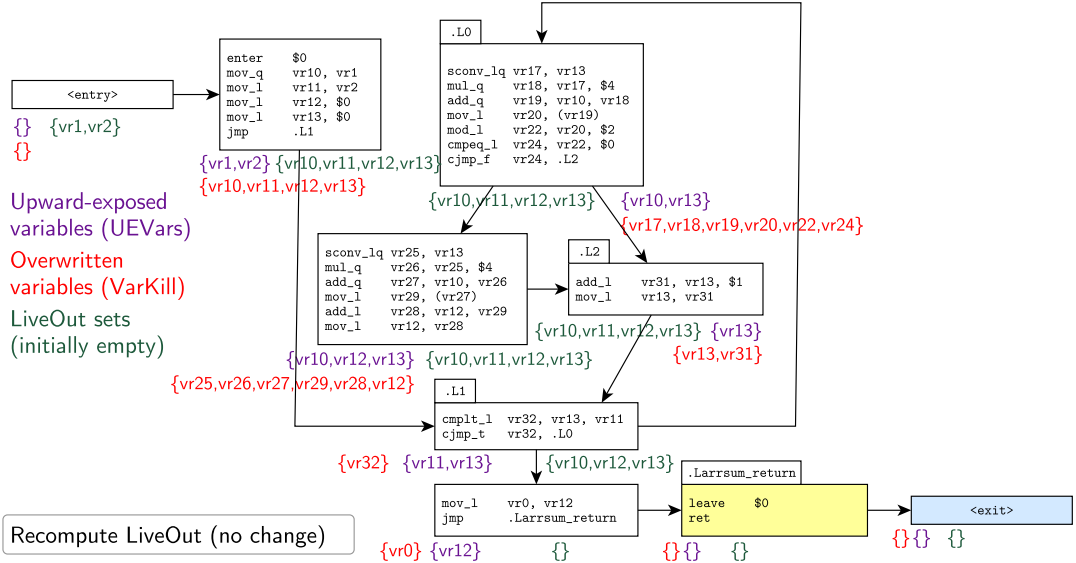
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (round 1 end)

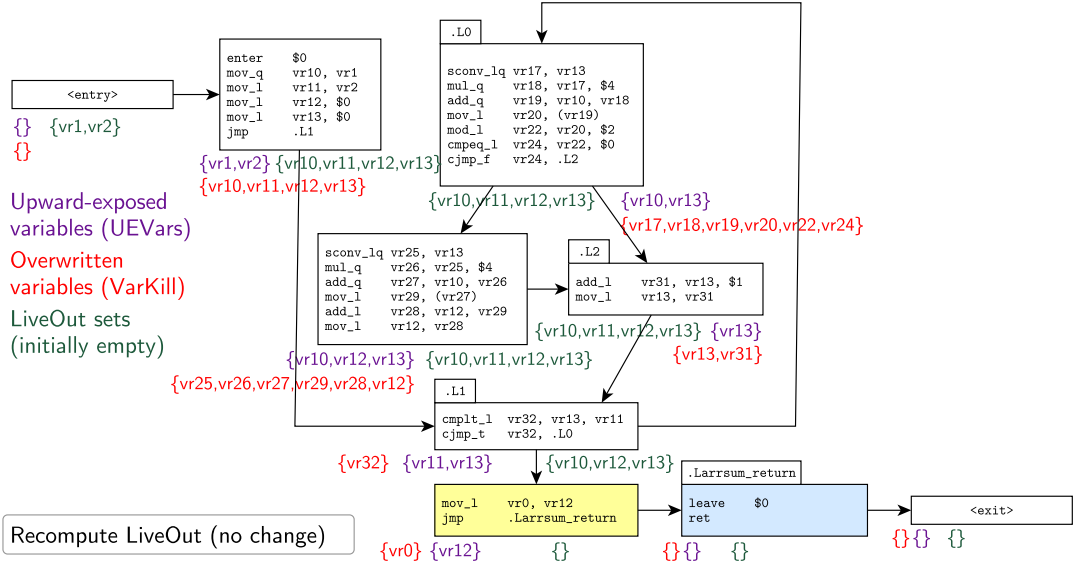
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (round 2 start)

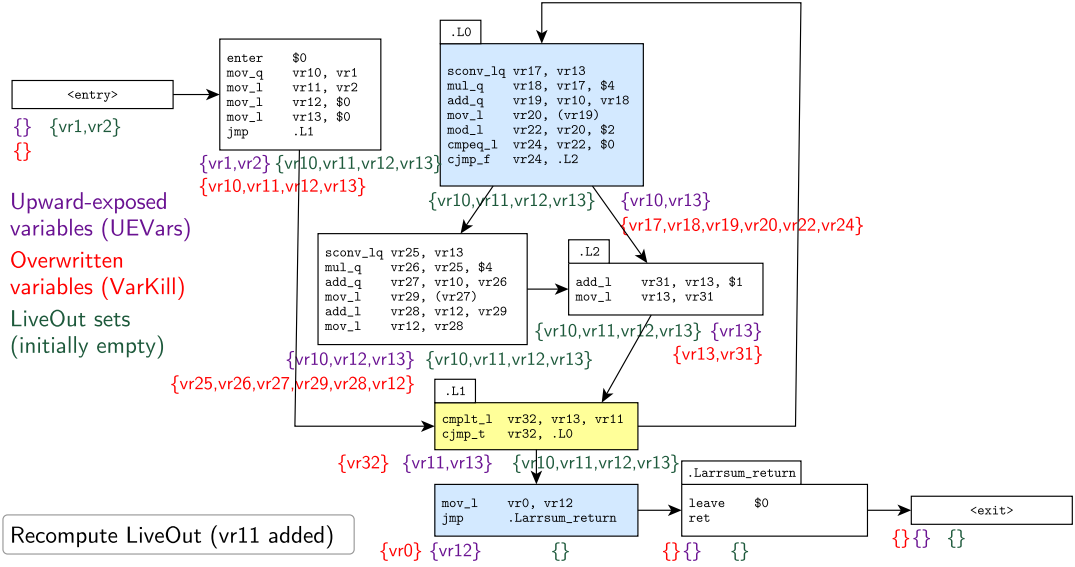
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (no change)

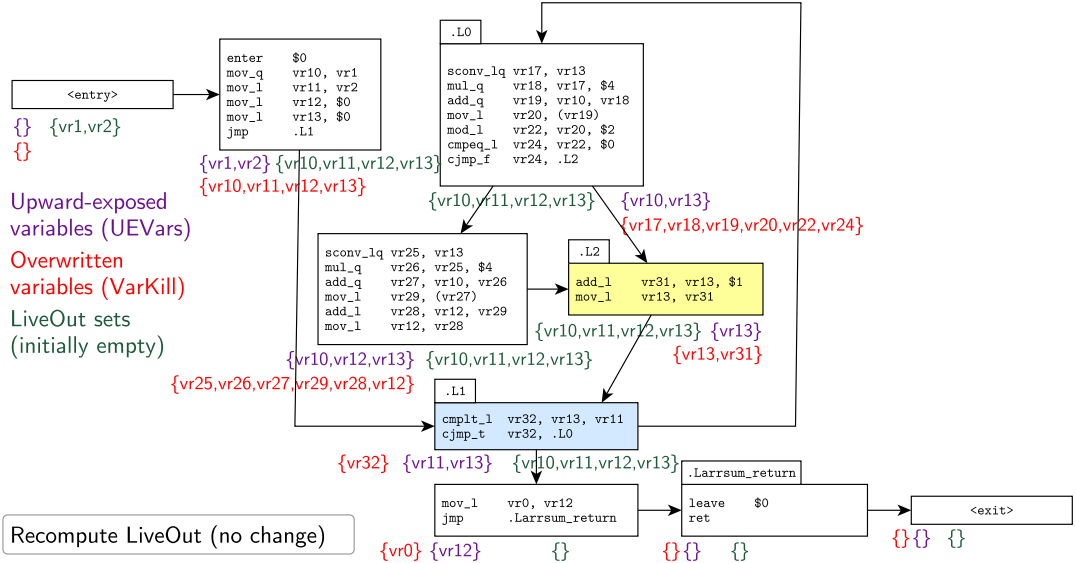
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13}
{vr10,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {}
{}{} {}
{}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (no change)

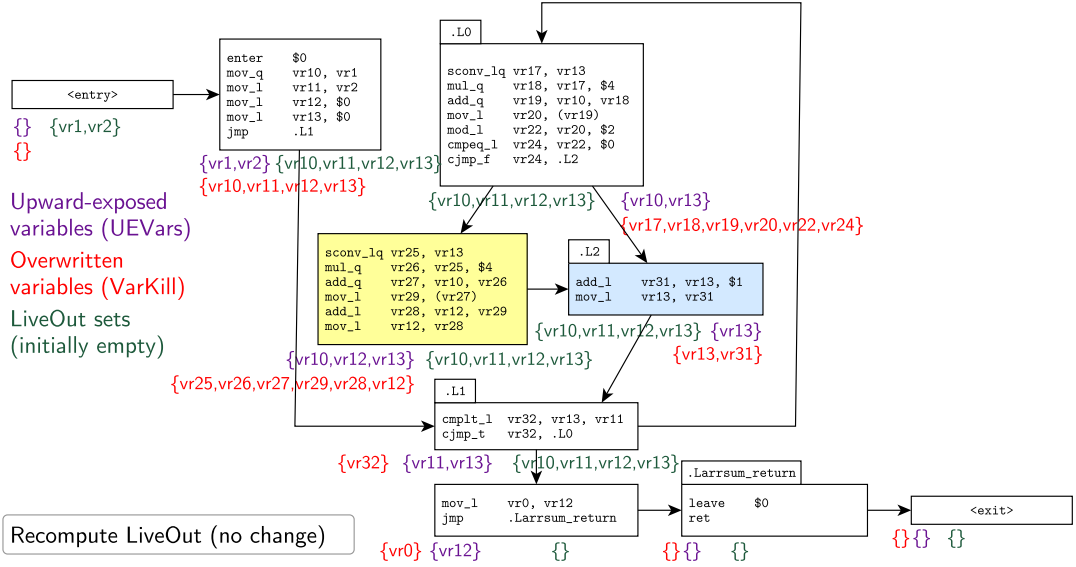
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {vr10,vr11,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (vr11 added)

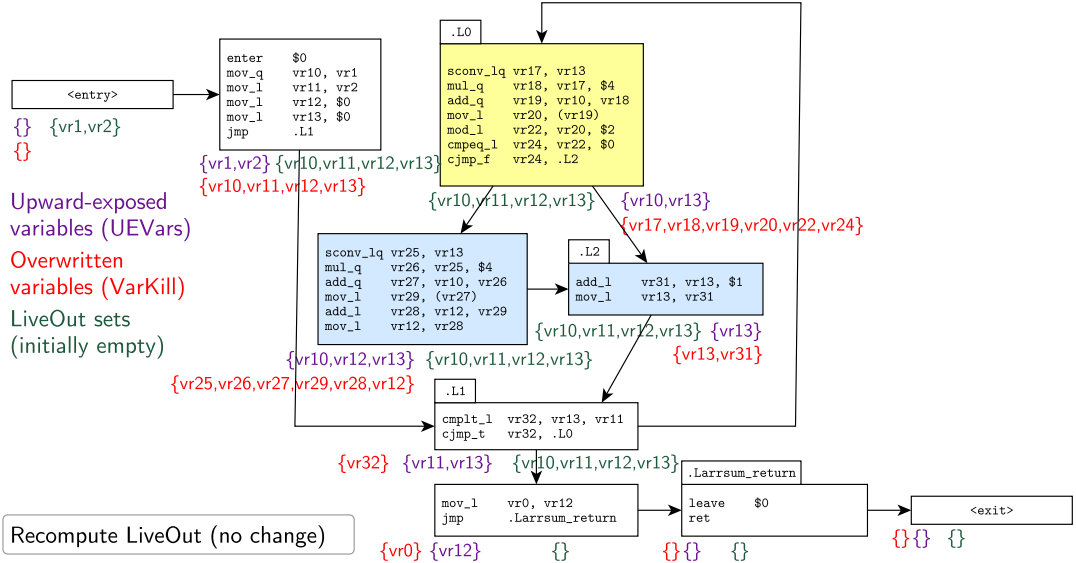
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {vr10,vr11,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (no change)

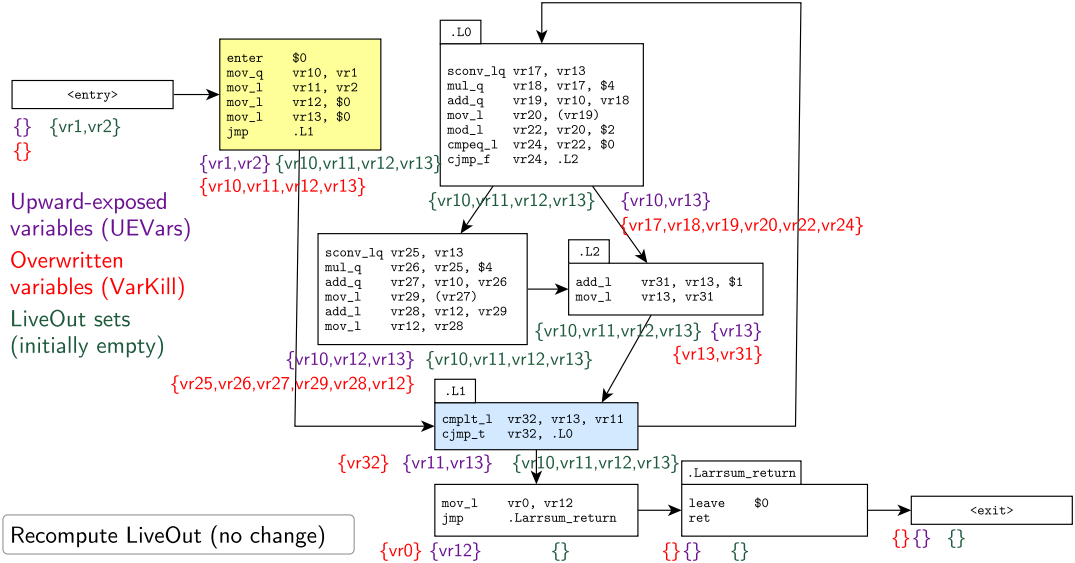
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {vr10,vr11,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (no change)

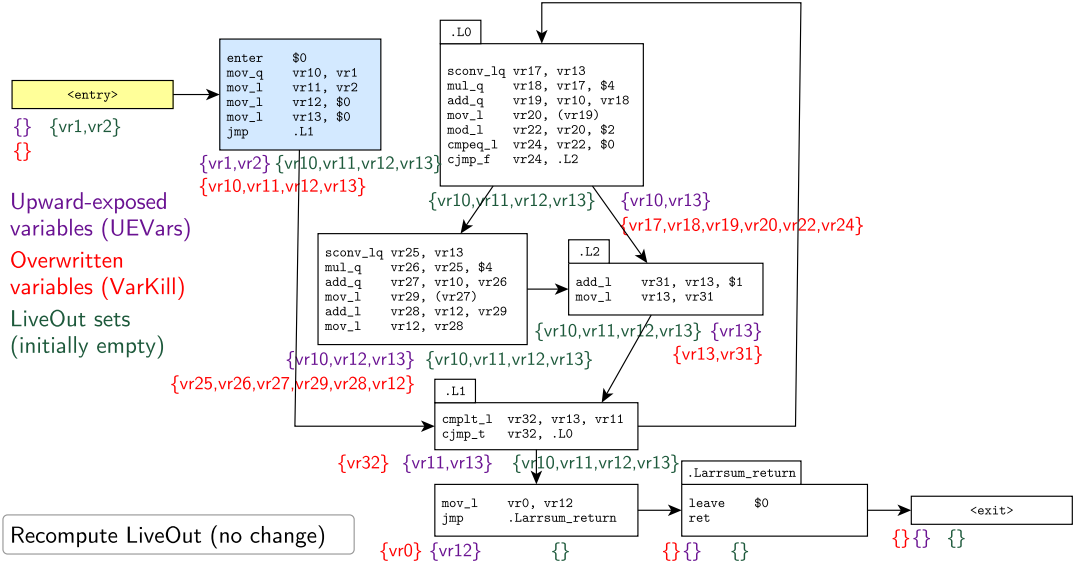
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {vr10,vr11,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Recompute LiveOut (no change)

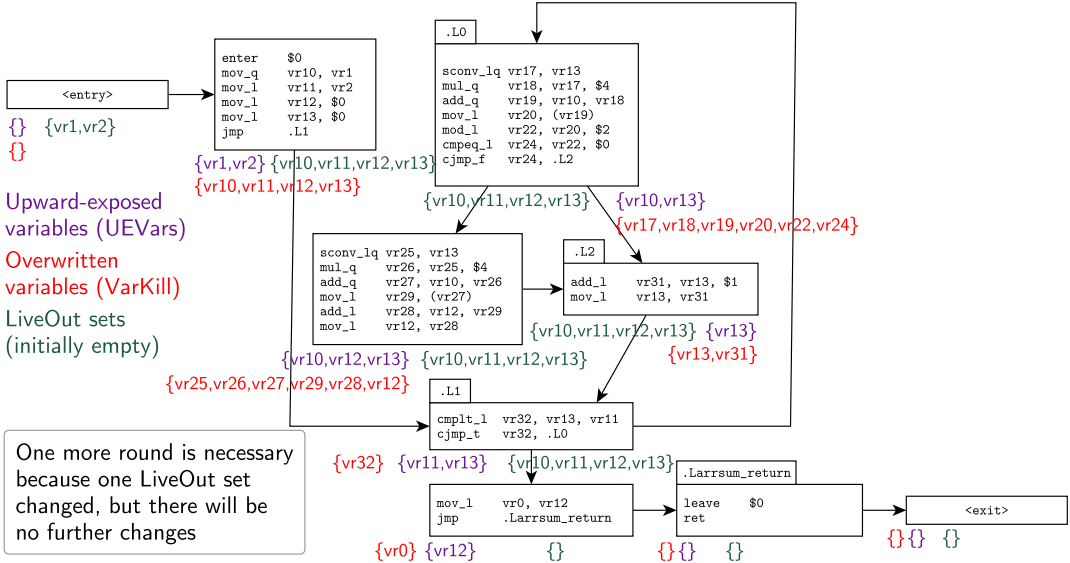
Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {vr10,vr11,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (no change)

Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13} {vr13}
{vr13,vr31}
{vr10,vr12,vr13} {vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32} {vr11,vr13} {vr10,vr11,vr12,vr13}
.Larrsum_return
mov_l vr0, vr12
jmp .Larrsum_return
leave $0
ret
<exit>
{vr0} {vr12} {} {}{} {} {}{} {}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
Recompute LiveOut (no change)

Example


<entry>
{} {vr1,vr2}
{}
enter $0
mov_q vr10, vr1
mov_l vr11, vr2
mov_l vr12, $0
mov_l vr13, $0
jmp .L1
{vr1,vr2} {vr10,vr11,vr12,vr13}
{vr10,vr11,vr12,vr13}
Upward-exposed variables (UEVars)
Overwritten variables (VarKill)
LiveOut sets (initially empty)
One more round is necessary because one LiveOut set changed, but there will be no further changes
.L0
sconv_lq vr17, vr13
mul_q vr18, vr17, $4
add_q vr19, vr10, vr18
mov_l vr20, (vr19)
mod_l vr22, vr20, $2
cmpeq_l vr24, vr22, $0
cjmp_f vr24, .L2
{vr10,vr11,vr12,vr13}
{vr10,vr13}
{vr17,vr18,vr19,vr20,vr22,vr24}
sconv_lq vr25, vr13
mul_q vr26, vr25, $4
add_q vr27, vr10, vr26
mov_l vr29, (vr27)
add_l vr28, vr12, vr29
mov_l vr12, vr28
{vr10,vr12,vr13}
{vr10,vr11,vr12,vr13}
{vr25,vr26,vr27,vr29,vr28,vr12}
.L2
add_l vr31, vr13, $1
mov_l vr13, vr31
{vr10,vr11,vr12,vr13}
{vr13}
{vr13,vr31}
.L1
cmplt_l vr32, vr13, vr11
cjmp_t vr32, .L0
{vr32}
{vr11,vr13}
{vr10,vr11,vr12,vr13}
mov_l vr0, vr12
jmp .Larrsum_return
{vr0}
{vr12}
{}
.Larrsum_return
leave $0
ret
{}{}
{}
<exit>
{}{}
{}

InstructionSequence and ControlFlowGraph

InstructionSequence and ControlFlowGraph

`InstructionSequence`: represents a linear sequence of `Instructions` (high-level or low-level)

`ControlFlowGraph`: a graph of basic blocks

- A basic block is just an `InstructionSequence` with a little bit of additional information
- A branch or function call can only be the last instruction in the basic block
- Instructions that are a control flow target must always be the first instruction in a basic block
- Edges of graph represent control flow possibilities

Converting InstructionSequence to ControlFlowGraph

```
// High-level InstructionSequence to high-level CFG
std::shared_ptr<InstructionSequence> hl_iseq = /* ... */
auto hl_cfg_builder = ::make_highlevel_cfg_builder(hl_iseq);
std::shared_ptr<ControlFlowGraph> hl_cfg = hl_cfg_builder.build();

// Low-level InstructionSequence to low-level CFG
std::shared_ptr<InstructionSequence> ll_iseq = /* ... */
auto ll_cfg_builder = ::make_lowlevel_cfg_builder(ll_iseq);
std::shared_ptr<ControlFlowGraph> ll_cfg = ll_cfg_builder.build();
```

Converting ControlFlowGraph to InstructionSequence

```
// Works for either high-level or low-level CFG
std::shared_ptr<ControlFlowGraph> cfg = /* ... */
std::shared_ptr<InstructionSequence> iseq =
  cfg->create_instruction_sequence();
```

Computing liveness information for high-level IR

```
std::shared_ptr<ControlFlowGraph> hl_cfg = /* ... */;
LiveVregs live_vregs(hl_cfg);
live_vregs.execute();

// live_vregs now has liveness information for virtual registers
// used in basic blocks of the control flow graph
```

Making use of liveness information

The best way to make use of liveness analysis results (or results from any other dataflow analysis) is to derive a class from `ControlFlowGraphTransform`:

- Your derived class's constructor executes the liveness analysis (and potentially other dataflow analyses) on the `ControlFlowGraph`
- Override the `transform_basic_block` member function to implement a local (basic-block scope) code transformation
- Within `transform_basic_block`, you can use the analysis's `get_fact_before_instruction` and/or `get_fact_after_instruction` functions to get the dataflow fact at the location immediately before or after a specified instruction in the basic block
- For liveness analysis, the dataflow fact is a `std::bitset` containing the register numbers of live virtual or machine registers

A control flow graph transformation

```
class MyTransform : public ControlFlowGraphTransform {
private:
  LiveVregs m_live_vregs;
  // ...other analyses if needed...

public:
  MyTransform(std::shared_ptr<ControlFlowGraph> cfg);

  virtual std::shared_ptr<InstructionSequence>
    transform_basic_block(std::shared_ptr<InstructionSequence> orig_bb);
};
```

CFG transform: constructor

```
MyTransform::MyTransform(std::shared_ptr<ControlFlowGraph cfg)
  : ControlFlowGraphTransform(cfg)
  , m_live_vregs(cfg) {
  m_live_vregs.execute(); // compute vreg liveness
}
```

CFG transform: basic block transform

```
std::shared_ptr<InstructionSequence>
MyTransform::transform_basic_block(std::shared_ptr<InstructionSequence> orig_bb
  std::shared_ptr<InstructionSequence> result_iseq(new InstructionSequence());

  for (auto i = orig_bb->cbegin(); i != orig_bb->cend(); ++i) {
    Instruction *orig_ins = *i;

    // ...
    // Determine live vregs after instruction executes
    LiveVregs::FactType fact
      m_live_vregs.get_fact_after_instruction(orig_ins);
    // ...

    result_iseq->append(/* transformed instruction */);
  }

  return result_iseq;
}
```

For liveness on low-level code

`LiveMregs` computes liveness for machine registers. It works the same way as `LiveVregs`, except that the dataflow facts are bitsets of machine registers containing live values.

- The bitset values are the ordinal values of members of the `MachineReg` enumeration (i.e., `MREG_RAX`, etc.)

For further details...

See Assignment 5 for more details